P.G. I (MATHEMATICS)
SYLLABUS

		Full Marks
Paper I:	**Unit 1 - Abstract Algebra I**	**45**
	Unit 2 - Abstract Algebra II	**45**
Paper II:	**Unit 1 - Real Analysis I**	**45**
	Unit 2 - Real Analysis II	**45**
Paper III:	**Unit 1 - Topology I**	**45**
	Unit 2 - Topology II	**45**
Paper IV:	**Unit 1 - Complex Analysis I**	**45**
	Unit 2 - Complex Analysis II	**45**
Paper V:	**Unit 1 - Differential Geometry of Manifolds I**	**45**
	Unit2 - Differential Geometry of Manifolds II	**45**
	Assignments	**50**

Paper : I

Unit I : Abstract Algebra I

Homomorphisms and Isomorphism . Cauchy theorem and p-grouops. Sylow Group and Theorems. Normal and subnormal series. Composition series. Jordon-Holder theorem. Solvable groups. Nilpotent groups.

Canonical forms - Similarity of linear transformations Invariant subspaces. Reduction to triangular forms. Nilpotent transfromation. The primary decomposition theorem. Jordan blocks and Jordan forms. Cyclic modules. Simple modules. Semi simple modules. Free modules.

Unit II : Abstract Algebra II

Polynomial Ring, Principal ideal ring, Euclidean Domains, Unique Factorization Domain, Noetherian and Artinian Ring.

Field Extension : Algebraic and transcendental Extensions. Separable and Inseparable extensions. Perfect fields, Normal extensions. Finite fields. Primitive elements. Algebrically closed fields. Galois extensions. Fundamental theorem of Galois theory. Solutions of polynomial equations by radicals. Insolvability of the general equation of degree 5 by radicals.

Noetherian and Artinian modules and rings. Hilbert basis theorem. Wedderburn-Arfin theorem. Uniform modules, primary modules. Noether - Laskar Theorem.
Smith normal form over a principal ideal domain and rank.

Fundamental structure theorem for finitely generated modules over a principal ideal domain and its application.Rational canonical form. Generalised Jordon form over any field.

References:

1. I.N. Herstein, Topics in. Algebra, Wiley Eastern Ltd., New Delhi, 1975
2. P. B. Bhattacharyya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra (2nd edn),Camb.Univ. Press, Indian Edition , 1997.
3. M. Artin, Algebra, Perentice -Hall of India, 1991.
4. P.M. Cohn, Algebra, vols, I,II, & III, John Wiley & Sons, 1982, 1989, 1991.
5. N. Jacobson, Basic Algebra, vols. I & II, W. H. Freeman, 1980 (also published by Hindustan Publishing Company)
6. S. Lang. Algebra, 3rd edn. Addison-Weslley, 1993.
7. I.S. Luther and I.B.S. Passi, Algebra, vol. I- groups, vol.II-Rings, Narosa Publishing House (vol. 1-1996, vol.II-1999)
8. D. S. Malik, J. N. Modrdeson, and M. K. Sen, Fundamentals of Abstract Algebra, Mc Graw-Hill, International Edition, 1997.
9. K. B. Datta, Matrix and Linear Algebra, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
10. S. K. Jain, A. Gunawardena and P. B. Bhattacharyya, Basic Linear Algebra with MATLAB, Key College Publishing (Springer-Verlag), 2001.
11. S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice-Hall of India, 2000.
12. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999
13. I. Stweart, Galois Theory, 2nd edition, Chapman and Hall, 1989.
14. J.P. Escofier, Galois theory, GTM Vol.204, Springer, 2001.
15. T.Y. Lam, Lectures on Modules and Rings, GTM Vol. 189, springer-verlag

16. D.S. Passman, A Course in Ring Theory , Wadsworth and Brooks/Cole Advanced Books and Softwares, Pacific Groves, California,1991.

Paper II

Unit I : Real Analysis I

Definition and existence of Riemann-Stieltjes integral, Properties of the Integral, Integration and differentiation, the fundamental theorem of Calculus, integation of vector-valued functions Rectifiable curves.Rearrangements of terms of a series, Riemann's theorem.

Sequences and series of functions, pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weierstrass M-test, Abel's and Dirichlet's tests for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stieltjes integration, uniform convergence and differentiation, Weierstrass approximation theorem, Power series, uniqueness theorem for power series, Abel's and Tauber's theorem.

Lebesgue outer measure. Measurable sets. Regularity. Measurable functions. Borel and Lebesgue measurability. Non-measurable sets.

Integration of non-negative functions. The General integral. Integration of Series. Reimann and Lebesgue Integrals.
The Four derivatives. Functions of Bounded variation. Lebesgue Differentiation Theorem Differentiation and Integration.
Measures and outer measures, Extension of a measure. Uniqueness of Extension, Completion of a measure. Measure spaces. Integration with respect to a measure.
The L^p -spaces. Convex functions, Jensen's inequality. Holder and Minkowski inequalities Completeness of L^p, Convergence in Measure, Almost uniform convergence.

Unit II : Real Analysis II

Functions of several variables, linear transformations, Derivatives in an open subset of R^n, Chain rule, Partial derivatives, interchange of the order of differentiation, Derivatives of higher orders, Taylor's theorem, Inverse function theorem, Implicit function theorem, Jacobians, extremum problems with constraints, Lagrange's multiplier method, Differentiation of integrals, Partitions of unity, Differential forms, Stoke's theorem.

References:
1. Walter Rudin, Principles of Mathematical analysis (3rd edition) McGraw-Hill, Kogakusha, 1976, International student edition.
2. T. M. Apostol, Mathematical Analysis, Narosa Publishing House , New Delhi, 1985.
3. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc. New York, 1975
4. A. J. White, Real Analysis; an introduction, Addison-Wesley Publishing Co., Inc., 1968.
5. G. de Barra, Measure Theory and Integration, Wiley Eastern Limited, 1981.
6. E. Hewitt and K. Stromberg. Real and Abstract Analysis, Berlin , Springer, 1969.
7. P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, New Age International (P) Limited Published, New Delhi, 1986 (Reprint 2000).

8. I. P. Natanson,Theory of Functions of a Real Variable.Vol. I,Frederick Ungar Publishing Co. 1961.
9. H. L. Royden, Real analysis, Macmillan Pub. Co. Inc. 4th edn. N.Y. 1993.
10. Richard L. Wheeden and Antoni Zygmund, Measure and Integral: An introduction to Real Analysis , Marcel Dekker Inc. 1977.
11. J. H. Williamson, Lebesgue Integration, Holt Rinehart and Winston, Inc. N.Y. 1962.
12. A. Friedman, Foundations of Modern Analysis, Holt, Rinehart and Winston, Inc., NY , 1970
13. P. R. Halmos, Measure Theory, Van Nostrand, Princeton, 1950
14. T. G. Hawkins, Lebesgue's Theory of Integration: Its Origins and Development, Chelsea, NY., 1979.
15. R. G. Bartle, The Elements of Integration, John Wiley & Sons, Inc. NY, 1966.
16. Serge Lang, Analysis I & II , Addison-Wesley Publishing Co., Inc. 1969.
17. Inder K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, Delhi, 1997.
18. Walter Rudin, Real & Complex Analysis, Tata McGraw-Hill Publishing Co. Ltd. New Delhi,1966

Paper III

Unit I : Topology I

Countable and uncountable sets. Infinite sets and the Axiom of Choice. Cardinal number and its arithmetic. Schroeder-Bernstein theorem. Cantor's theorem and the continuum hypothesis. Zorn's lemma. Well-ordering theorem.
Definition and examples of topological spaces. Closed sets. Closure. Dense subsets. Neighbourhoods. Interior, exterior and boundary. Accumulation points and derived sets. Bases and sub-bases. Subspaces and relative topology.

Alternate methods of defining a topology in terms of Kuratowski Closure Operator and Neighbourhood Systems.

Continuous functions and homeomorphism.

First and Second Countable spaces. Lindelof's theorems. Separable spaces. Second Countability and Separability.

Separation axioms T_0, T_1, T_2 , $T_{3\frac{1}{2}}$, T_4 their Characterizations and basic properties. Urysohn's lemma. Tietz extension theorem.

Tychonoff product topology in terms of standard sub-base and its characterizations. Projection maps.

Unit II : Topology II

Compactness. Continuous functions and compact sets. Basic properties of compactness. Compactness and finite intersection property. Sequentially and countably compact sets. Local compactness and one point compactification. Stone-vech compactification. Compactness in metric spaces. Equivalence of compactness, countable compactness and sequential compactness in metric spaces.

Connected spaces. Connectedness on the real line. Components. Locally connected spaces.

Separation axioms and product spaces. Connectedness and product spaces. Compactness and product spaces (Tychonoff's theorem). Countability and product spaces.

Embedding and metrization. Embedding lemma and Tychonoff embedding. The Urysohn metrization theorem.

Nets and filters. Topololgy and convergence of nets. Hausdorffness and nets. Compactness and nets. Filters and their convergence. Canonical way of converting nets to filters and vice-versa. Ultra-filters and compactness.
Metrization theorems and Paracompactness-Local finiteness. The Nagata-Smirnov metrization theorem. Paracompactness. The Smirnov metrization theorem.

The fundamental group and covering spaces-Homotopy of paths. The fundamental group. Covering spaces. The fundamental group of the circle and the fundamental theorem of algebra.

References:

1. James R, Munkres, Topology, A first course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. J. Dugundji, Topology, Allyn and Bacon, 1966 (Reprinted in India by Prentice Hall of India Pvt.Ltd.
3. George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Book Compoany, 1963f.
4. K.D. Joshi, Introduction to General Topology, Wiley Eastern Ltd., 1983.
5. J. Hocking and G. Young, Topology, Addison-Weslley, Reading, 1961
6. J. L. Kelley, General Topology, Van Nostrand, Reinhold Co., NY, 1995.
7. L. Steen and J. Seebach, counter examples in topology, Holt, Rinehart and Winston, New York, 1970.
8. W. Thron, Topological Structures, Holt, Rinehart and Winston, New York, 1966.
9. N. Bourbaki, General Topology, Part I (Transl). Addison Wesley, Reading, 1966.
10. R. Engelking, General Topology, Polish Scientific Publishers, Warszawa, 1977.
11. W. J. Pervin, Foundations of General Topology, Academic Press Inc. NY. 1964.
12. E. H. Spanier, Algebraic Topology. Academic Press Inc. NY, 1966.
13. S. Willard, General Topology, Addison-Wesley, Reading , 1970.
14. Crump W. Baker, Introduction to Topology, Wm C. Brown Publisher, 1991
15. Sze-Tsen Hu, Elements of General Topology, Holden-Day. Inc. 1965.
16. D. Bushaw, Elements of General Topology, John Wiley & Sons, NY, 1963.

Paper IV

Unit I : Complex Analysis I

Complex integration. Cauchy-Goursat. Theorem. Cauchy's integral formula. Higher order derivatives. Morera's Theorem. Cauchy's inequality and Liouville's theorem. The fundamental theorem of algebra. Taylors theorem. Maximum modulus principle. Schwarz lemma. Laurent's series. Isolated singularities. Meromorphic functions. The argument principle. Rouche's theorem inverse function theorem.

Residues. Cauchy's residue theorem. Evaluation of integrals. Branches of many valued functions with special reference to arg z , log z and z^a.

Bilinear transformations, their properties and classifications. Definitions and examples of Conformal mappings.

Analytic continuation. Uniqueness of direct analytic continuation. Uniqueness of analytic continuation along a curve. Power series method of analytic continuation

Unit II: Complex Analysis II

Spaces of analytic functions. Hurwitz's theorem. Montel's theorem Riemann mapping theorem.

Weierstrass' factorisation theorem. Gamma function and its properties. Riemann Zeta function. Riemann's functional equation. Runge's theorem. Mittag-Leffler's theorem. Schwarz Reflection Principle.Monodromy theorem and its consequences. Harmonic functions on a disk. Harnack's inequality and theorem. Dirichlet problem. Green's function.
Canonical prolducts Jensen's formula. Poisson-Jensen formula. Hadamard's three circles theorem. Order of an entire function. Exponent of Convergence of zeros. Borel's theorem. Hadamard's factorization theorem.
The range of an analytic function. Bloch's theorem. The Little Picard theorem. Schottky's theorem. Montel Caratheodory and the Great picard theorem.
Univalent functions. Bieberbach's conjecture (Statement only) and the "$^1/_4$- theorem.

Reference:

1. H. A. Priestly, Introduction to Complex Analysis, Clarendon Press Oxford, 1990
2. J. B. Conway, Functions of one Complex variable. Springer-Verlag. International student-edition, Narosa Pub. House. 1980.
3. Liang-shin Hahn & Bernard Epstein, Classical Complex Analysis. Jones and Bartlett Pub. International London, 1996.
4. L. V. Ahlfors. Complex Analysis, McGraw
5. S. Lang. Complex Analysis, Addison Wesley. 1970.
6. D. Sarason, Complex Function Theory , Hindustan Book Agency, Delhi, 1994
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12. Walter Rudin, Real and Complex Analysis, McGraw - Hill Book Co, 1966.
13. S. Saks and A. Zygmund, Analytic Functions, Monografie Matematyczne, 1952
14. E. C. Titchmarsh, The Theory of Functions, Oxford Univ. Press, London.
15. W. A. Veech, A Second Course in Complex Aanlysis. W. A. Benjamin, 1967.
16. S. Ponnusamy, Foundations of Complex Analysis, Narosa Pub. House, 1997.

Paper V

Unit I : Differential Geometry of Manifolds I

Definition and examples of Differentiable manifolds. Tangent Spaces. Jacobian map. One parameter group of transformations. Lie derivatives. Immersions and Imbeddings. Distributions. Exterior Algebra, Exterior Derivative.

Topological groups. Lie groups and lie algebras. Product of two Lie groups. One parameter subgroups and exponential maps. Examples of lie groups. Homomorphism and Isomorphism. Lie transformation groups. General linear groups. Principal fibre bundle. Linear frame bundle. Associated fibre bundle. Tangent bundle. Induced bundle. Bundle homomorphisms.

Unit II : Differential Geometry of Manifolds II

Riemannian manifolds. Riemannian connection. Curvature tensors. Sectional Curvature. Schur's theorem. Geodesics in a Riemannian manifold. Projective curvatrure tensor. Conformal curvature tensor.

Submanifolds & Hypersurfaces. Normals. Gauss. Weingarten equation. Lines of curvature. Generalized Gauss and Maniardi - Codazi equations.

Almost Complex manifolds. Nijenhuis tensor. Contravariant and covariant almost analytic Vector fields. F-connection.

References

1. R. S. Mishra, A course in tensors with applications to Riemannian geometry, Pothishala (pvt) Ltd., 1965.
2. R. S. Mishra, Structures on a differentiable manifold and their applications, Chandra Prakashan, Allahabad, 1984.
3. B. B. Sinha, An Introduction to Modern Differential Geometry, Kalyani Publishers, New Delhi,1982
4. K. Yano and M. Kon, Structure of Manifolds, World Scientific Pub. Co. Pvt. Ltd. 198