M.Sc Semester –III (2019-2020)

Assignment

Subject- Mathematics

Course –Linear Algebra

Subject Course No.-DEMATH3CORE1

Total Marks-25

Answer any one of the following questions (15 marks)

1. Let V, W, and Z be vector spaces, and let $T: V \to W$ and $U: W \to Z$ be linear.

   (a) Prove that if UT is one-to-one, then T is one-to-one. Must U also be one-to-one ?

   (b) Prove that if UT is onto, then U is onto. Must T also be onto ?

   (c) Prove that if U and T are one-to-one and onto, then UT is also.

2. Define Symmetric Bilinear form. Let V be a finite dimensional vector space over a field F not of characteristic two. Then prove that every symmetric bilinear form on V is diagonalizable.

Answer any one of the following questions (10 marks)

1. Prove that a linear operator T on a finite dimensional vector space V is diagonalizable if and only if the minimal polynomial of T is of the form $p(t) = (t - \lambda_1)(t - \lambda_2) \dots \dots (t - \lambda_k)$, where λ_1, λ_2, ….., λ_k are the distinct eigen values of T.

2. Find the Jordan canonical form B of A and matrix $Q \in M_{3\times 3}(\mathbb{R})$ such that $Q^{-1}AQ = B$ where $A = \begin{pmatrix} 0 & 1 & -1 \\ -4 & 4 & -2 \\ -2 & 1 & 1 \end{pmatrix}$.

M.Sc Semester –III (2019-2020)

Assignment

Subject- Mathematics

Course –Functional Analysis

Subject Course No.-DEMATH3CORE2

Total Marks-25

<u>Answer any one of the following questions (15 marks)</u>

1. (a) If z is any fixed element of an inner product space X, show that $f(x) = < x, z >$ defines a bounded linear functional f on X, of norm $\|z\|$.

(b) Show that the dual space of the real space l^2 is l^2.

2. State and prove the open mapping Theorem and then deduce the closed graph Theorem from it.

Answer any one of the following questions (10marks)

1. Show that two Hilbert spaces H and $\widetilde{H}$, both real or both complex, are isomorphic if and only if they have the same Hilbert dimension.
2. Let $T: \mathcal{D}(T) \to Y$ be a bounded linear operator, where $\mathcal{D}(T)$ lies in normed space X and Y is Banach space. Then show that T has an extension $\check{T}: \overline{\mathcal{D}(T)} \to Y$ where $\check{T}$ is a bounded linear operator of the form $\|\check{T}\| = \|T\|$.

M.Sc Semester –III (2019-2020)

Assignment

Subject- Mathematics

Course –Partial Differential Equations

Subject Course No.-DEMATH3SCORE3

Total Marks-25

<u>Answer any one of the following questions (15 marks)</u>

1. Use Duhamel's principle to solve the heat equation problem described by

$$u_t(x,t) = ku_{xx}(x,t) + f(x,t), \qquad -\infty < x < \infty, \qquad t > 0$$

$$u(x,0) = 0, -\infty < x < \infty.$$

2. Solve the initial value problem of the wave equation, described by the inhomogeneous wave equation

$$u_{tt} - c^2 u_{xx} = f(x,t)$$

subject to $\quad u(x,0) = \eta(x), \quad u_t(x,0) = \nu(x).$

Answer any one of the following questions (10marks)

1. Reduce the following equation to canonical form and hence solve it.

$$x^2(y-1)u_{xx} - x(y^2-1)u_{xy} + y(y-1)u_{yy} + xyu_x - u_y = 0.$$

2. Find the deflection $u(x,t)$ of a taut string which was at rest at time $t = 0$, if it is fastened at the end point $x = l$ and subjected at the other end point $x = 0$ to a motion represented by $u(0,t) = f(t)$.

M.Sc Semester –III (2019-2020)

Assignment

Subject- Mathematics

Course –Discrete Mathematics

Subject Course No.-DEMATH3OLEC4

Total Marks-25

Answer any one of the following questions (15 marks)

1. (a) What is the generating function for a sequence of numbers $\{a_0, a_1, \dots\dots, a_n, \dots\}$? Use it compute the number of n-digit quarternary sequences that have an even numbers of 1's.
   (b) Determine the coefficient of $x^5y^{10}z^5w^5$ in $(x - 7y + 3z - w)^{23}$.
   (c) What is pigeonhole principle ? The integers from 1 to 10 are randomly distributed around a circle. Using pigeonhole principle, prove that there must be three-neighbours whose sum is at least 17.
2. (a) Using principle of inclusion and exclusion count the number of permutations of $\{1,2, \dots\dots n\}$ without fixed points.

(b) Check whether the statement is true or false with proper justification: The number of spanning tress of the complete graph with n vertices is n^{n-2}.

(c) Let G be a graph with n vertices. Prove that the following statements are equivalent:

(i) G is tree.

(ii) G is connected and has $n-1$ edges.

(iii) Any two vertices of G are connected by exactly one path.

Answer any one of the following questions (10marks)

1. (a) Let $G(V, E)$ be a simple planer graph with $V = \{1,2, \ldots\ldots, n\}$. Prove that $min\{d(i)|\ i = 1, 2, \ldots, n\} \leq 5$, where $d(i)$ is the degree of vertex i.

   (b) Let $G(V, E)$ be a simple graph and contains a vertex of degree 3. Verify: G is Eulerian.

   (c) Let $G(V, E)$ be a simple graph with n vertices. Prove that G has at least two vertices with same degree.

2. (a) Define a planer graph, show that K_5 is a non-planer.

(b) Using the generating function, solve the difference equation

$$y_{n+2} - y_{n+1} - 6y_n = 0, y_1 = 1, y_0 = 2.$$

M.Sc Semester –III (2019-2020)

Assignment

Subject- Mathematics

Course– Elementary Number Theory

Subject Course No.-DEMATH3OLEC5

Total Marks-25

<u>Answer any one of the following questions (15 marks)</u>

1. State and prove the Möbius inversion formula. Then show that for any positive integer n, $\varphi(n) = n\sum_{d|n}\frac{\mu(d)}{d}$, where d runs through the positive divisors of n.
2. State and prove Euler's theorem. Find the units digit of 3^{100} by means of Euler's theorem.

Answer any one of the following questions (10marks)

1. (a) If is a p prime, then prove that $(p-1)! \equiv -1(mod\ p)$. Is the converse of the statement is true ? ......Justify.

   (b) Prove that there are infinite number of primes of the form $4n+3$.

1. Let a be an odd integer. Then show that the following holds:

   (a) $x^2 \equiv a(mod\ 2)$ always has a solution.

   (b) $x^2 \equiv a(mod\ 4)$ has a solution if and only if $a \equiv 1(mod\ 4)$.

   (c) $x^2 \equiv a(mod\ 2^n)$, for $n \geq 3$, has a solution if and only if $a \equiv 1(mod\ 8)$.