M.Sc Semester –IV (2018-2019)

Assignment

Subject- Mathematics

Course –Abstract Measure Theory

Subject Course No.-DEMATH4CORE1

Total Marks-25

Answer any one of the following questions (15 marks)

1. Let μ be a measure on a σ-algebra $\mathcal{A}$ of subset of a set X.
   (a) For an arbitrary sequence$\{E_n : n \in \mathbb{N}\}$ in $\mathcal{A}$, we have $\mu\left(\lim_{n\to\infty} inf\ \ E_n\right) \leq \lim_{n\to\infty} inf\, \mu(E_n)$. Give an example where the equality doesn't hold.
   (b) (b) If there exists $A \in \mathcal{A}$ with $\mu(A) < \infty$ such that $E_n \subset A$ for all $n \in \mathbb{N}$ then $\mu\left(\lim_{n\to\infty} \sup E_n\right) \geq \lim_{n\to\infty} sup\ \mu(E_n)$. Give an example where the equality doesn't hold.
   (c) If $\lim_{n\to\infty} E_n$ exists and if there exists $A \in \mathcal{A}$ with $\mu(A) < \infty$ such that $E_n \subset A$ for all $n \in \mathbb{N}$ then $\lim_{n\to\infty} \mu(E_n)$ exists and $\mu\left(\lim_{n\to\infty} E_n\right) = \lim_{n\to\infty}\ \mu(E_n)$.

2. Let $(X, \mathcal{A})$ be a measurable space and $\{f_n : n \in \mathbb{N}\}$ monotone sequence of extended real valued $\mathcal{A}$-measurable functions on a set $D \in \mathcal{A}$. Prove that $\lim_{n\to\infty} f_n$ exists on D and is $\mathcal{A}$-measurable on D.

Answer any one of the following questions (10 marks)

1. Suppose that f is any extended real-valued function which for every x and y satisfies $f(x) + f(y) = f(x + y)$.
   (i) Show that f is either everywhere finite or everywhere infinite. (ii) Show that if f is measurable and finite, then $f(x) = xf(1)$ for each x.
2. Let μ be a measure on a σ-algebra $\mathcal{A}$ of subset of a set X. If $\{E_n : n \in \mathbb{N}\}$ is a decreasing sequence in $\mathcal{A}$ and if $\mu(E) < +\infty$, then $\mu(\bigcap_{n=1}^{\infty} E_n) = \lim_{n\to\infty} \mu(E_n)$. Show that equality may fail if the finiteness condition $\mu(E) < +\infty$, is dropped.

M.Sc Semester –IV (2018-2019)

Assignment

Subject- Mathematics

Course –Numerical problem solving by computer programming

Subject Course No.-DEMATH4SCORE2

Total Marks-25

Answer any one of the following questions (15 marks)

1. (a) Why is an array called a data structure ? What is a dynamic array ? How is it created ?

   (b) Write a program to read a matrix of size $m \times n$ and print its transpose.

1. Compare, in terms of their functions, the following pairs of statements:

   (a) While and do…while. (b) While and for.

   (c) Break and goto. (d) Break and continue.

   (e) Continue and goto.

Answer any one of the following questions (10marks)

1. Distinguish between the following:

   (a) Global and local variables. (b) Automated and static variables. (c) Initialization and assignment of variables.

2. Write a program in ANSI C to compute integration of $f(x) = e^{x tan x}$ from 0 to 0.8 by Simpson's $1/3^{rd}$ rule taking number of intervals $n = 8$.

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M.Sc Semester –IV (2018-2019)

Assignment

Subject- Mathematics

Course –Integral Equations and Integral transform

Subject Course No.-DEMATH4ELEC4

Total Marks-25

<u>Answer any one of the following questions (15 marks)</u>

1. (a) Obtain the radius of convergence of the Neumann series when the functions $f(x)$ and $K(x,t)$ are continuous function on the interval (a,b).

(b) Solve the integral equation $y(x) = f(x) + \lambda \int_0^1 (x+t)y(t)dt$ and find the eigenvalues.

2. Find the Fourier transform of $f(t) = \begin{cases} 1, & |t| < a \\ 0, & |t| > a \end{cases}$

And, hence prove that $\int_0^\infty \frac{sin^2(at)}{t^2} dt = \frac{\pi a}{2}$.

Answer any one of the following questions (10marks)

1. Find resolvent kernel and then solve the following integral equation $y(x) = 1 + \lambda \int_0^\pi \sin(x+t)\, y(t)dt$.

2. (a) Compute the displacement $u(x,t)$ of an infinite string using the method of Fourier transform given that the string is initially at rest and that the initial displacement is $f(x), -\infty < x < \infty$.
(b) Find the Laplace transform of $\sin\sqrt{t}$.

M.Sc Semester –IV (2018-2019)

Assignment

Subject- Mathematics

Course –Field extension and Galois Theory

Subject Course No.-DEMATH4ELEC5

Total Marks-25

<u>Answer any one of the following questions (15 marks)</u>

1. (a) Prove that there is always an algebraically closed field extension K of given field F.
   (b) Let $F \subseteq L \subseteq K$be a tower of field. Prove that K / F is an algebraic extension if and only if K / L and L / F are algebraic extensions.
2. (a) Let E / F be a Galois extension and K be intermediate subfield of E / F. Then prove that K / F is Galois if and only if $G(E/K) \Delta G(E / F)$.

(b) Let K be a splitting field of $x^4 - 2$ over $\mathbb{Q}$. Find the Galois group $G(K \,/\, \mathbb{Q})$ and draw the lattice structure of its subgroups.

Answer any one of the following questions (10marks)

1. (a) Prove that for any prime p, there exists a field extension $F/\mathbb{Z}_p$ of arbitrary finite degree n.
   (b) Construct a field with 9 elements.
2. (a) Let K be a field of characteristic $p > 0$. Prove that K is perfect if and only if $K = K^p$.
   (b) Consider the field extension $\mathbb{R}/\mathbb{Q}$. Show that $\pi - 3$ is transcendental over $\mathbb{Q}$.

M.Sc Semester –IV (2018-2019)

Assignment

Subject- Mathematics

Course– Algebraic Topology

Subject Course No.-DEMATH4ELEC6

Total Marks-25

Answer any one of the following questions (15 marks)

1. Prove that the fundamental group of the product space is isomorphic to the product of fundamental groups of factors.
2. (a) Show that the fundamental group of the figure eight is not abelian.
   (b) Let $\mathcal{C}$ be a simple closed curve in $\mathbb{S}^2$. Prove that $\mathcal{C}$ separates $\mathbb{S}^2$.

Answer any one of the following questions (10marks)

1. Let X, Y be two path-connected spaces which are of the same homotopy type. Prove that their fundamental groups are isomorphic.

2. Prove that the fundamental group of $\mathbb{S}^1$ is isomorphic to the additive group $\mathbb{Z}$ of integers.

M.Sc Semester –IV (2018-2019)

Assignment

Subject- Mathematics

Course– General theory of Integration

Subject Course No.-DEMATH4ELEC7

Total Marks-25

Answer any one of the following questions (15 marks)

3. Let $f: [a, b] \to \mathbb{R}$ be measurable, let A and B be measurable subsets of $[a, b]$, and let f be a Henstock integrable on A and B.
   (a) Suppose that $A \subseteq B$. Prove that let f be a Henstock integrable on $B - A$.
   (b) Suppose that A and B are disjoint. Prove that let f be a Henstock integrable on $A \cup B$.
   (c) Show that f be a Henstock integrable on $B - A$.
4. (a) Let $I = [a, b]$ is a non-degenerate compact interval in $\mathbb{R}$ and δ is a gauge on I. Prove that there exists a δ-fine partition of I.
   (b) Show that null function is Henstock integrable and integral value is zero.

Answer any one of the following questions (10marks)

1. (a) State and prove Saks-Henstock Lemma.

   (b) Let$f: [a, b] \to \mathbb{R}$ has a primitive F on. Prove that f is Henstock integral on $[a, b]$ and $(HK)\int_a^b f = F(b) - F(a)$.
2. State and prove Squeez Theorem.