M.Sc Semester –III (2018-2019)

Assignment

Subject- Mathematics

Course –Linear Algebra

Subject Course No.-DEMATH3CORE1

Total Marks-25

<u>Answer any one of the following questions (15 marks)</u>

1. (a) Let V be a vector space and $T: V \rightarrow V$ be a linear transformation. Prove that if the minimal polynomial of T is of the form $p(t) = (\varphi(t))^m$, then there exists a rational canonical basis for T.
   (b) Find the rational canonical form $C$of A and matrix $Q \in M_{2\times 2}(\mathbb{R})$ such that $Q^{-1}AQ = C$ where $A = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$.
2. Define inner product space. Prove that every non-zero finite dimensional inner product space V has an orthonormal basis β. Furthermore, if $\beta = \{v_1, v_2, \dots\dots v_n\}$ and $x \in V$, then $x = \sum_{i=1}^{n} \langle x, v_i \rangle v_i$.

<u>Answer any one of the following questions (10 marks)</u>

1. Prove that a linear operator T on a finite dimensional vector space V is diagonalizable if and only if V is the direct sum of the eigen spaces of T.
2. State and prove Cayley-Hamilton Theorem for a linear operator.

M.Sc Semester –III (2018-2019)

Assignment

Subject- Mathematics

Course –Functional Analysis

Subject Course No.-DEMATH3CORE2

Total Marks-25

Answer any one of the following questions (15 marks)

1. State and prove Hahn-Banach Theorem.
2. State and prove the open mapping Theorem. Then deduce the closed graph Theorem.

Answer any one of the following questions (10marks)

1. Show that a normed vector space is finite dimensional if and only if its closed unit ball is compact.
2. If f is a non-zero linear functional on an infinite dimensional linear space X, does there exist a norm on X such that f is discontinuous?

M.Sc Semester –III (2018-2019)

Assignment

Subject- Mathematics

Course –Partial Differential Equations

Subject Course No.-DEMATH3SCORE3

Total Marks-25

Answer any one of the following questions (15 marks)

1. Find the solution of the wave equation representing the vibration of a circular membrane.
2. **2.** Define the Neumann problem for a rectangle and solve it.

Answer any one of the following questions (10marks)

1. Reduce the following equation to canonical form and hence solve it

$$(1 + x^2)u_{xx} + (1 + y^2)u_{yy} + xu_x + yu_y = 0.$$

2. Find the temperature in a sphere of radius a, when its surface is kept at zero temperature and its initial temperature is $f(r, \theta)$.

M.Sc Semester –III (2018-2019)

Assignment

Subject- Mathematics

Course –Discrete Mathematics

Subject Course No.-DEMATH3ELEC4

Total Marks-25

Answer any one of the following questions (15 marks)

1. State and prove Generalized pigeonhole principle. Give an example of it.
2. (a) State and prove principle of inclusion and exclusion for two sets.
   (b) Let A and B be two finite sets such that: n (A-B) =25, n (A∪ B) =100, n (A∩ B) =40, find n (B).

Answer any one of the following questions (10marks)

1. State and prove fundamental theorem of semi group homomorphism.
2. Define a planer graph, show that K_5 is a non-planer.

M.Sc Semester –III (2018-2019)

Assignment

Subject- Mathematics

Course– Elementary Number Theory

Subject Course No.-DEMATH3ELEC5

Total Marks-25

Answer any one of the following questions (15 marks)

1. If p is a prime and $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots .. a_1 x + a_0$, $a_n \not\equiv 0\ (mod\ p)$ is a polynomial of degree $n \geq 1$ with integral coefficients, then prove that the congruence $f(x) \equiv 0 (mod\ p)$ has atmost n incongruent solutions modulo p.
2. State and prove Fermat's theorem. Using Fermat's theorem prove that, if p is an odd prime, then $1^p + 2^p + \cdots \ldots + (p-1)^{p-1} \equiv 0 (mod\ p)$.

Answer any one of the following questions (10marks)

1. Prove that the linear Diophantine equation $ax + by = c$ has a solution if and only if $d|c$, where $\mathrm{d} = \gcd(\mathrm{a}, \mathrm{b})$. Furthermore if x_0, y_0 is any particular solution of this equation, then all other solutions are given by $x = x_0 + (\frac{b}{d})t$ and $y = y_0 - (\frac{a}{d})t$, where t is an arbitrary integer.
2. Prove that every positive integer $n > 1$ can be expressed as a product of primes; this representation is unique, apart from the order in which the factors occur.