M.Sc Semester –II (2019-2020)

Assignment

Subject- Mathematics

Course –Real Analysis

Subject Course No.-DEMATH2CORE1

Total Marks-25

Answer any one of the following questions (15 marks)

1. State and prove Monotone Convergence theorem for sequence of measurable sets.
2. Let μ^* is a metric outer measure on a metric space (X, d). Let $(A_n : n \in \mathbb{N})$ be an increasing sequence in $\mathfrak{B}(X)$ and $A = \lim_{n\to\infty} A_n$. If A_n and $A_n \setminus A_{n+1}$ are positively separated for every $n \in \mathbb{N}$, then $\mu^*(A) = \lim_{n\to\infty} \mu^*(A_n)$.

Answer any one of the following questions (10 marks)

1. Let E be the set of all irrational numbers in the interval $(0,1)$. Show that for every $\varepsilon \in (0,1)$ there exists a closed set C in $\mathbb{R}$ such that $C \subset E$ and $\mu_L(C) > 1 - \varepsilon$.

2. Let μ^* be a metric outer measure on a set X. Show that a non- μ^* mesurable subset of X exists if and only if μ^* is not countably additive on $\mathfrak{B}(X)$.

M.Sc Semester –II (2019-2020)

Assignment

Subject- Mathematics

Course –Point set Topology

Subject Course No.-DEMATH2CORE2

Total Marks-25

<u>Answer any one of the following questions (15 marks)</u>

1. Let X be a metrizable space. Then prove that following are equivalent –
   (1) X is compact.
   (2) X is limit point compact.
   (3) X is sequentially compact.
2. Define a metrizable space. If $\mathcal{A}$ is an open covering of a metrizable space X, then prove that there is an open covering $\boldsymbol{\mathcal{C}}$ of X refining A that is countably locally finite.

<u>Answer any one of the following questions (10marks)</u>

1. Prove that the arbitrary product of compact space is compact in the product topology.
2. Let X be a completely regular space. Then prove that there exists a compactification Y of X having the property that every bounded continuous map $f: X \to \mathbb{R}$ extends uniquely to a continuous map of Y into $\mathbb{R}$.

M.Sc Semester –II (2019-2020)

Assignment

Subject- Mathematics

Course –Ordinary Differential Equations

Subject Course No.-DEMATH2SCORE3

Total Marks-25

Answer any one of the following questions (15 marks)

1. Define fundamental matrix. Let $A(t)$ be a continuous $n \times n$ matrix defined on interval I and let φ satisfy $x^{'} = Ax$. Then show that φ satisfy the first order equation $(\det\varphi)^{'} = (tr\ A)\det\varphi$, which is equivalent to $\det\varphi(t) = \det\varphi(\tau)\int_{\tau}^{t} tr(\varphi(s))ds$.
2. Obtain the power series solution of the Legendre equation $(1 - t^2)x^{''} - 2tx^{'} + p(p+1) = 0$ where p is a constant. Deduce that the power series solution of the Legendre equation is the sum of a polynomial of degree p and a power series, when p is a positive integer.

Answer any one of the following questions (10marks)

1. Define Legendre polynomials. State and prove Rodrique's formula.
2. Derive the generating function of the Hermite polynomials. Using the generating function prove the following $(i) H_{2n+1}(0) = 0\ and\ (ii)\ H_{2n}(0) = (-1)^n \dfrac{2n!}{n!}$.

M.Sc Semester –II (2019-2020)

Assignment

Subject- Mathematics

Course –Theory of Rings and Modules

Subject Course No.-DEMATH2ELEC4

Total Marks-25

Answer any one of the following questions (15 marks)

1. Define Artinian ring with example. State and prove Cohen's theorem.
2. Let $E, E^{'}$ be modules and assume that $E^{'}$ is free. Let $f: E \to E^{'}$ be a surjective homomorphism. Then show that there exists a free submodule F of E such that the restriction of f to F induces an isomorphism of F to $E^{'}$ and also $E = F \oplus ker\ f$.

Answer any one of the following questions (10marks)

1. State and prove Hilbert basis theorem.
2. Let M be a finitely generated free module over R, a PID and let $X = \{e_1, e_2, \dots., e_n\}$ be a basis of M. Show that if N a submodule of M then N is also free and has rank at most n elements.

M.Sc Semester –II (2019-2020)

Assignment

Subject- Mathematics

Course– Complex Analysis II

Subject Course No.-DEMATH2ELEC5

Total Marks-25

Answer any one of the following questions (15 marks)

1. State and prove Jensen's formula for the closed unit disk.
2. Let f be an entire function of finite genus μ, then prove that f is of finite order λ and $\lambda \leq \mu + 1$.

Answer any one of the following questions (10marks)

1. Suppose that $a_k \geq 0$ for all $k \in \mathbb{N}$. Then prove that the series $\sum_{k=1}^{\infty} a_k$ converges absolutely if and only if the series $\sum_{k=1}^{\infty} Log(1 + a_k)$ converges absolutely.
2. State and prove Harnack's Principle/ Theorem for sequence of harmonic functions.