M.Sc Semester –I (2020-2021)

Assignment

Subject- Mathematics

Course –Abstract Algebra

Subject Course No.-DEMATH1CORE1

Total Marks-25

Group A

Answer **any one** of the following questions (15 marks)

1. (a) State and prove Sylow's second Theorem.
   (b) Show that if G is a group of order 168 that has a normal subgroup of order 4, then G has a normal subgroup of order 28.
2. (a) Let D be an integral domain. Then prove that there exists a field F that contains a subring isomorphic to D.
   (b) Prove that the ideal $< x >$ in $\mathbb{Q}[x]$ is maximal.

Group B

Answer **any one** of the following questions (10 marks)

1. (a) Let $f(x) \in \mathbb{Z}[x]$. Prove that if $f(x)$ is reducible over $\mathbb{Q}$, then it is reducible over $\mathbb{Z}$.
   (b) Determine all ring isomorphism from $\mathbb{Z}_n$ to itself.

2. (a) Prove that for every positive integer n, $Aut(\mathbb{Z}_n)$ is isomorphic to $U(n)$.

(b) Suppose that $\varphi: \mathbb{Z}_{20} \to \mathbb{Z}_{20}$ is a group automorphism and $\varphi(5) = 5$. What are the possibilities for $\varphi(x)$?

M.Sc Semester –I (2020-2021)

Assignment

Subject- Mathematics

Course –Complex Analysis I

Subject Course No.-DEMATH1CORE2

Total Marks-25

Group A

Answer **any one** of the following questions (15 marks)

1. (a) Suppose z_0 is an essential singularity of $f(z)$. Then show that for every complex number w_0, there is a sequence $z_n \to z_0$ such that $f(z_n) \to w_0$.

(b) Show that if $f(z)$ is an entire function, and there is a nonempty disk such that $f(z)$ does not attain any values in the disk, then $f(z)$ is constant.

2. (a) State and prove Rouch$\acute{e}$'s Theorem.

(b) Suppose that f is meromorphic on $\mathbb{C}_\infty$. Then prove or disprove the following: f has a pole only at ∞ if and only if $Res[f(z), \infty] = 0$.

Group B

Answer **any one** of the following questions (10 marks)

1. (a) A function $f(z)$ is rational if and only if it is meromorphic in the extended complex plane $\mathbb{C}_\infty$.

   (b) Prove or disprove the following: The point $z = 0$ is the only singularity of the function $f(z) = \sin\left(1 - {}^{1}\!/_{z}\right)$ and $z = 0$ is a simple pole.

2. (a) Prove that a bounded entire function is constant.

   (b) Suppose that f is continuous on a domain D and analytic on $D\backslash\{z_0\}$ for some $z_0 \in D$. Then show that f is analytic on D.

M.Sc Semester –I (2020-2021)

Assignment

Subject- Mathematics

Course –Analysis of Several Variables

Subject Course No.-DEMATH1SCORE3

Total Marks-25

Group A

Answer **any one** of the following questions (15 marks)

1. State and prove Implicit function theorem in $\mathbb{R}^n$.
2. Prove that the function $f: M_n(\mathbb{R}) \to \mathbb{R}$ defined by $f(A) = \det A$, $\forall A \in M_n(\mathbb{R})$ is differential and $Df(I)X = Tr\ (X),\ \forall X \in M_n(\mathbb{R})$ where $Tr(X) = Trac\ of\ X =$sum of the diagonal elements.

Group B

Answer **any one** of the following questions (10 marks)

1. Derive Stoke's theorem and give an example to verify the theorem.

2. (a) Derive a necessary and sufficient condition for the integrability of a bounded function $f(x, y)$ over a rectangle $R = \{a \leq x \leq b; c \leq y \leq d\}$.

(b) Let $H \in M_n(\mathbb{R})$, then prove that $\lim_{H \to 0} \frac{|\det H|}{\|H\|} = 0$.

M.Sc Semester –I (2020-2021)

Assignment

Subject- Mathematics

Course –Differential Geometry

Subject Course No.-DEMATH1ELEC4

Total Marks-25

Group A

Answer **any one** of the following questions (15 marks)

1. (a) Prove that a circular helix is a Bertrand curve.
   (b) Find the curvature and torsion at any point of a given curve $\mathcal{C}$, given by $\mathcal{C}: x^1 = a, x^2 = t, x^3 = ct$, where $ds^2 = (dx^1)^2 + (x^1)^2(dx^2)^2 + (dx^3)^2, a, c$ are non-zero constant.

2. (a) Prove that $\kappa = 0$ is the necessary and sufficient condition for a surface to be a developable.
   (b) Show that the geodesic curvature of the curve $u = c$ on a surface with metric $\varphi^2(du)^2 + \mu^2(dv)^2$ is $\frac{1}{\phi\mu}\frac{\partial\mu}{\partial u}$.

Group B

Answer **any one** of the following questions (10 marks)

1. (a) Find the differential equations for the geodesic in cylindrical coordinate.
   (b) Prove that the Gaussian curvature is an invariant.
2. (a) Derive the equation of Weingarten.
   (b) Show that the normal curvatures in the directions of the co-ordinate curves are $\frac{b_{11}}{a_{11}}$ and $\frac{b_{22}}{a_{22}}$.

M.Sc Semester –I (2020-2021)

Assignment

Subject- Mathematics

Course– p-adic Analysis

Subject Course No.-DEMATH1ELEC5

Total Marks-25

Group A

Answer **any one** of the following questions (15 marks)

1. (a) Prove the set of p-adic integers Z_p is a subring of Q_p.
   (b) Show every element of Z_p is the limit of a sequence of (non-negative) integers and conversely. Also prove that every Cauchy sequence in Q consisting of integers as a limit in Z_p.
   2. Prove that $C(\mathbb{Z}_p)$ is a ring with non-Archimedean seminorm $\| \|_p$. Moreover, show that $C(\mathbb{Z}_p)$ is complete with respect to this seminorm.

Group B

Answer **any one** of the following questions (10 marks)

1. (a) Let $a \in Q_p$, $m \in Z$. Then prove that $B(a, p^{-m})$ is both open and compact in the p-adic topology.
   (b) Show that p-adic digit expansion is unique.
2. (a) Let $f: \mathbb{Z}_p \to \mathbb{Q}_p$ be locally constant. Then prove that image of f, $im\ f = f(\mathbb{Z}_p) = \{f(\alpha): \alpha \in \mathbb{Z}_p\}$, is a finite set.
   (b) Prove the Hensel's Lemma for many variables and functions.