M.Sc Semester –I (2020-2021)

[From January 2021]

Assignment

Subject- Mathematics

Course –Abstract Algebra

Subject Course No.-DEMATH1CORE1

Total Marks-25

Group A

Answer **any one** of the following questions (15 marks)

1. (a) Let G be a finite group and p be a prime. If p^k divides order of G, then show that G has at least one subgroup of order p^k.
   (b) Show that no group of order p^2q^2, where p, q are prime integers, is simple.
2. (a) Prove that in an integral domain every prime element is irreducible but converse is not true in general. Under what condition converse holds? --- Justify your answer.
   (b) Show that $\mathbb{Z}_3[x]/< x^2 + x + 1 >$ is not a field.

Group B

Answer **any one** of the following questions (10 marks)

1. (a) Show that there are only two non-isomorphic rings of p elements, where p is prime integer.
   (b) Is the ring $\mathbb{Q}[\sqrt{2}]$ isomorphic to the ring $\mathbb{Q}[\sqrt{3}]$?.
2. (a) Let H_1, H_2 be subgroups of a group G with H_2 normal in G. Then prove that ${H_1}/{(H_1 \cap H_2)}$ is isomorphic to ${(H_1 H_2)}/{H_2}$.
   (b) Suppose that $\varphi: \mathbb{Z}_{50} \to \mathbb{Z}_{50}$ is a group automorphism and $\varphi(11) = 13$. What are the possibilities for $\varphi(x)$?

M.Sc Semester –I (2020-2021)

[From January 2021]

Assignment

Subject- Mathematics

Course –Complex Analysis I

Subject Course No.-DEMATH1CORE2

Total Marks-25

Group A

Answer **any one** of the following questions (15 marks)

1. (a) Suppose that f is continuous on a domain D and analytic on $D\backslash\{z_0\}$ for some $z_0 \in D$. Then show that f is analytic on D.

   (b) If S, the set of zeros of an analytic function f in a domain D, has a limit point z^* in D. Then prove that $f(z) \equiv 0$ in D.

2. (a) If f is analytic within and on a positively oriented simple closed contour γ and does not vanish on γ, then show that $\frac{1}{2\pi i}\int \frac{f^{(1)}(z)}{f(z)} dz = N - P$, over γ where N, P are number of zeros

and poles of f which lies inside γ, both of zeros and poles being counted according to their multiplicity.

(b) Prove that the equation $z^3 + iz + 1 = 0$ has one root in each of the first, second and fourth quadrants.

<u>Group B</u>

Answer **any one** of the following questions (10 marks)

1. Let $f(z)$ be analytic in a domain D containing a segment of the x-axis and be symmetric to that axis. Then prove that $\overline{f(z)} = f(\bar{z}),\ z \in D$ if and only if $f(x)$ is real for each point on the segment of x-axis.
2. Suppose z_0 is an essential singularity of $f(z)$. Then show that for every complex number w_0, there is a sequence $z_n \to z_0$ such that $f(z_n) \to w_0$.

M.Sc Semester –I (2020-2021)

[From January 2021]

Assignment

Subject- Mathematics

Course –Analysis of Several Variables

Subject Course No.-DEMATH1SCORE3

Total Marks-25

Group A

Answer **any one** of the following questions (15 marks)

1. State and prove Inverse function theorem in $\mathbb{R}^n$.

2. Let $f: \mathbb{R}^n \to \mathbb{R}^m$be differential at $a \in \mathbb{R}^n$ and $g: \mathbb{R}^m \to \mathbb{R}^p$be differential at $f(a) \in \mathbb{R}^m$, then show that $g \circ f: \mathbb{R}^n \to \mathbb{R}^p$be differential at $a \in \mathbb{R}^n$ and $D(g \circ f)(a) = Dg\big(f(a)\big) \circ Df(a)$.

Group B

Answer **any one** of the following questions (10 marks)

1. Derive Gauss (Divergence) theorem and give an example to verify the theorem.
2. (a) Derive a necessary and sufficient condition for the integrability of a bounded function $f(x, y)$ over a rectangle $R = \{a \leq x \leq b; c \leq y \leq d\}$.

   (b) Let S be an open connected subset of $\mathbb{R}^n$, and let $f: S \rightarrow \mathbb{R}^m$ be differentiable at each point of S. If $f^{(1)}(c) = 0$ for each c in S, then prove that f is constant on S.

M.Sc Semester –I (2020-2021)

[From January 2021]

Assignment

Subject- Mathematics

Course –Differential Geometry

Subject Course No.-DEMATH1ELEC4

Total Marks-25

Group A

Answer **any one** of the following questions (15 marks)

1. (a) Find the necessary and sufficient condition for two curves $\Gamma: y^i = y^i(s)$ and $\bar{\Gamma} = \bar{y}^i = \bar{y}^i(s) = y^i(s) + a(s)\mu^i(s)$are Bertrand.

   (b) Investigate the spherical image of a circular helix.

2. (a) Prove that $\kappa = 0$ is the necessary and sufficient condition for a surface to be a developable.

   (b) Show that the first fundamental form for the surface $x^1 = u^1 \cos u^2\,, x^2 = u^1 \sin u^2\,, x^3 = 0$, is given by $ds^2 = (du^1)^2 + (u^1)^2(du^2)^2$.

<u>**Group B**</u>

Answer **any one** of the following questions (10 marks)

1. Show that the geodesic of the sphere of radius a determined by the equation $y^1 = a\cos u^1 \cos u^2$, $y^2 = a\cos u^1 \sin u^2$, $y^3 = a\sin u^1$, are great circles.
2. (a) Derive the equation of Weingarten.

   (b) Show that the normal curvatures in the directions of the co-ordinate curves are $\frac{b_{11}}{a_{11}}$ and $\frac{b_{22}}{a_{22}}$.

M.Sc Semester –I (2020-2021)

[From January 2021]

Assignment

Subject- Mathematics

Course– p-adic Analysis

Subject Course No.-DEMATH1ELEC5

Total Marks-25

Group A

Answer **any one** of the following questions (15 marks)

1. Prove that $C(\mathbb{Z}_p)$ is a ring with non-Archimedean seminorm $\| \|_p$. Moreover, show that $C(\mathbb{Z}_p)$ is complete with respect to this seminorm.
2. Prove that there is a unique sequence of locally constant, hence continuous functions $\omega_n : \mathbb{Z}_p \longrightarrow \mathbb{Q}_p$ satisfying

    (i) $\omega_n(\alpha)^p = \omega_n(\alpha)$ for $n \geq 0$,

$(ii) \alpha = \sum_{n=0}^{\infty} \omega_n(\alpha) p^n$.

Group B

Answer **any one** of the following questions (10 marks)

1. (a) Prove that if R is a field with norm N then $\hat{R}$ is a field. In particular, $\mathbb{Q}_p$ is a field.

   (b) Prove that $\{x_n\}$ is a Cauchy sequence in $\mathbb{Q}_p$ if and only if $\{x_{n+1} - x_n\}$ is a null sequence.

2. (a) Show that p-adic digit expansion is unique .

   (b) Let $f: \mathbb{Z}_p \to \mathbb{Q}_p$ be locally constant. Then prove that image of f, $im\ f = f(\mathbb{Z}_p) = \{f(\alpha): \alpha \in \mathbb{Z}_p\}$, is a finite set.